Fact and Fallacy in EEG Source Connectivity

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There has been very early work with functional Connectomics using EEG.

A more modern example plotting Imaginary Coherence between 60 EEG electrodes during the resting state seems to indicate complex dynamics?

Consider this resting state (alpha rhythm coherence matrix with Scalp (60 electrodes). Does it represent an interesting complex neural network?

Actually NO
Outline

• What is the Effective Brain Connectome?
• Why formulate the EBC is a State Space Model?
• Are there magical measures that find EBC without solving the EEG inverse problem?
• Does the situation change by previously finding components?
• What would be the ground truth?
• What are some of the remaining Challenges?
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In Brain Connectivity there are related but different concepts that are confused.

- Anatomical (axonal) Connectivity
- Effective (causal) Connectivity
- Direct Causal (hormonal) effect
- Functional Connectivity = Correlation (Non causal)
- Functional Connectivity is not connectivity!!!
The Effective Brain Connectome (EBC) is the set of all Effective Brain Networks.

An Effective Brain Network is a (hyper) graph:

\[ E \subseteq \Psi \times \Psi \times T \rightarrow A \]

- **E**: Effective Brain Network graph
- **\Psi**: Activity at nodes
- **T**: Delays
- **A**: Effective Connectivity measure: Wiener-Akaike-Granger Schweder (WAGS)

Points (neural masses) and Manifolds (neural fields) are included in the network.
One WAGS measure is Granger Causality based on the Bivariate Autoregressive Model

\[
\psi_{1,t} = a_{11} \psi_{1,t-1} + a_{12} \psi_{2,t-1} + e_{1,t}
\]

\[
\psi_{2,t} = a_{21} \psi_{1,t-1} + a_{22} \psi_{2,t-1} + e_{1,t}
\]

\(t = 1, \ldots, N\)

Granger Non Causality

\(H_0 : a_{21} = 0 \iff I_{2 \rightarrow 1} = 0\)

This is the basis of all connectivity measures
Multivariate Granger Causality is a test of whether the autoregressive coefficients are zero.

\[
\begin{align*}
\psi_{1,t} & = a_{1,1} \psi_{1,t-1} + a_{1,2} \psi_{2,t-1} + \cdots + a_{1,N_s} \psi_{N_s,t-1} + e_{1,t} \\
\psi_{2,t} & = a_{2,1} \psi_{1,t-1} + a_{2,2} \psi_{2,t-1} + \cdots + a_{2,N_s} \psi_{N_s,t-1} + e_{2,t} \\
& \vdots \\
\psi_{N_s,t} & = a_{N_s,1} \psi_{1,t-1} + a_{N_s,2} \psi_{2,t-1} + \cdots + a_{N_s,N_s} \psi_{N_s,t-1} + e_{N_s,t}
\end{align*}
\]

\( t = 1, \ldots, N \)

\( N_s \quad t \quad t-1 \)

1 2 ... Ns
For the autoregressive Matrices the columns correspond to emitters and the rows to receivers.
The MAR model has a compact mathematical expression which we will use in the rest of the talk.

\[ \psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \]
The Effective Brain Connectome is a 3 dimensional Tensor

A Vector is a 1-D tensor

\[ \mathbf{x} = \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(I) \end{bmatrix} \]

A Matrix is a 2-D tensor

\[ \mathbf{X} = \begin{bmatrix} x(1,1) & \ldots & x(1,J) \\ \vdots & \ddots & \vdots \\ x(I,1) & \ldots & x(I,J) \end{bmatrix} \]

An example of a 3-D tensor

\[ \mathbf{X} = \begin{bmatrix} x(1,1,1) & \ldots & x(1,J,1) \\ \vdots & \ddots & \vdots \\ x(I,1,1) & \ldots & x(I,J,1) \end{bmatrix} \]
The Effective Brain Connectome is a 3 dimensional Tensor

$E \subset \Psi \times \Psi \times T \rightarrow A$

$(\psi, \psi, \tau) \rightarrow a$

The support of the marginal graph

$\Psi \times \Psi$

$(\psi, \psi)$

Is the anatomical connectome
Outline

• What is the Effective Brain Connectome?
• **Why formulate the EBC is a State Space Model?**
• Are there magical measures that find EBC without solving the EEG inverse problem?
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There is much current research about different types of Brain Connectivity:
Effective Brain Connectome must be estimated via a State-Space Model

\[ \psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \]  
\[ \nu_t = K \psi_t + \epsilon_t \]

State Evolution Equation

Observation Equation
Effective Brain Connectome must be estimated via a State-Space Model.

\[
\begin{align*}
\psi_t &= \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \\
\nu_t &= K \psi_t + \varepsilon_t
\end{align*}
\]

Observation Equation  
State Evolution Equation
Outline

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This must be done using Bayesian inversion of the state space model with the appropriate priors.

\[
\psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t
\]

\[
v_t = K \psi_t + \varepsilon_t
\]
A common mistake is to attempt to estimate effective connectivity from the observations ignoring the observation equation. Instead of estimating \( A, \psi \) from \( v_t \), estimate \( A, \psi \) from \( v_t \) by means of a “magical Measure”.
A common mistake is to attempt to estimate effective connectivity from the observations ignoring the observation equation.

**State Evolution Equation**

\[ \psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \]

**Observation Equation**

\[ \nu_t = K \psi_t + \varepsilon_t \]
Refutations of “magical Measures” has recently been published

**Critical Comments on EEG Sensor Space Dynamical Connectivity Analysis**

Frederik Van de Steen¹ · Luca Faes² · Esin Karahan³ · Jitkomut Songsrit⁴ · Pedro A. Valdes-Sosa⁵ · Daniele Marinazzo¹

*frontiers in Computational Neuroscience*

**Volume Conduction Influences Scalp-Based Connectivity Estimates**

Clemens Brunner¹,² · Martin Billinger¹ · Martin Seeber³,⁴ · Timothy R. Mullen³ and Scott Makeig⁴
Why this won't work can be easily seen examining the formulas.

\[
\psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t
\]

State Evolution Equation

\[
\nu_t = K \psi_t + \epsilon_t
\]

Observation Equation

Imply that

\[
\nu_t = K \sum_{k=1}^{N_k} A_k \psi_{t-1} + \xi_t
\]
There is no way for volume conduction NOT to affect the MAR as supported by simulations.
Defenders of the magical measures show simulations in which there is no correct conclusions are obtained in spite of volume conduction.

Unfortunately examples don’t refute general theoretical arguments.

However a SINGLE example does refute a general claim.

Mistrust any measure which is no obtained by using the lead Field!!!!!
At the heart of many magical proposals is the “phase fallacy”

It argues that measures based on phase or lag are not affected by volume conduction.

For example, some argue that looking at the imaginary component of the signal in the frequency domain is unaffected by volume conduction.

Quote: The lead field is a real matrix therefore does not affect phase.
Such is the argument that misinterprets Imaginary Coherence, which is easily shown to be false.

Remember the model for observations:

$$v_t = K \sum_{k=1}^{N_k} A_k \psi_{t-1} + \xi_t$$

Transforms in the frequency domain to

$$v_f = KH(f) \psi_f + \xi_f$$

$$H(f) = \sum_{k=1}^{N_k} A_k e^{-i2\pi fk}$$
There is no way that Volume conduction is eliminated by taking the imaginary part of the signal!!!!!!!!!!

\[
\text{Im } \nu_f = K \text{ Im} \left( H(f) \psi_f \right) + \text{Im } \xi_f
\]

Taking the Fourier transform does not change the physics of the problem.
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EEG sources may also be found in the frequency domain.

128 channel EEG recording

Descriptive EEG Spectral parameters

Source Spectra found using VARETA

Topographic Maps

Tomography
Another approach is to do Component Analysis

\[ \text{EEG} \begin{array}{c} v_s \\ \text{voxel} \end{array} = \sum_k q_k \begin{array}{c} b_k \\ \text{time} \end{array} \]

Require these to be either:

- Orthogonal PCA
- nearly independent ICA
Component Analysis is a powerful method for analyzing functional connectivity

- Given the EEG data, $X$, we would like to decompose it into source scalp maps multiplied by source activity, $X = AS$, with $A$ and $S$ unknown
ICA extracts components by assuming independence

\[ \sum_{k} q_{k} b_{k} \]

Typical ICA scalp maps

Typical ICA sources – alpha

Tomography
STONNICA is ICA tomography not tomography of ICA carried out in the source space by assuming nonoverlapping sources

\[ v = K \cdot M \cdot \psi + e \]

- \( L \cdot M \) must be minimum in the L1 norm.
- \( M \) must also be column orthogonal and nonnegative.
A different type of Component results from recognizing that EEG time/frequency data is a tensor
Spatio Temporal Non Negative ICA (STONNICA) can be extended to T/F with a TENSOR model.
The real answer to the network question of resting alpha is actually a bit simpler than some would want. For components one must also solve the inverse problem.
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In the EEG state space model estimation of sources and connectivity intertwined

**Observation Equation**

\[ v_t = K \psi_t + \varepsilon_t \]

**State Evolution Equation**

\[ \psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \]
Effective Brain Connectome must be estimated via a State-Space Model

\[ \psi_t = \sum_{k=1}^{N_k} A_k \psi_{t-1} + e_t \]

Observation Equation

\[ v_t = K \psi_t + e_t \]

State Evolution Equation

Inverse solution first and then estimating connectivity is not optimal
Connectivity methods should be tested in experimental situations. One such set of data is available for simultaneous EEG/ECoG in a Macaque preparation.

Joint EEG (10/20 system) and ECoG (128 channels)

Forward head model (Valdes-Hernandez) is to be used to test surface EEG estimated effective connectivity against actual ECoG effective connectivity.
ECoG Connectivity in this situation is directly measurable
And in fact shows considerable correspondence with the EEG recorded simultaneously as shown by Canonical Correlation Analysis.

Canonical Correlations between EEG and ECog
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  • Improving Source connectivity estimates
  • Dealing with wave phenomena
The EEG spectrum can be separated out into two component founds in the late 80's

A number of authors in the 80’s had found consistent Principal components for EEG topographies

We speculated they might be Spherical Harmonics thus reflecting a cortical random isotropic random process on the cortical surface.

\[ Y = \{ Y_{lm} \} \]

Neural field theory has revived Spherical Harmonic expansions

Eigenmodes of brain activity: Neural field theory predictions and comparison with experiment

P.A. Robinson, X. Zhao, K.M. Aquino, J.D. Griffiths, S. Sarkar, Grishma Mehta-Pandejee
There is a deep connection between neural field theory and “Microstates”

“inverse solution” of microstates in terms of space/time frequency
Modeling the cortex as a spherical sheet allows the expansion of sources in terms of spherical harmonics

\[ g(t) = Y \tilde{g}(t) \]
\[ p(t) = Y \tilde{p}(t) \]

The inverse Laplacian can be expressed also in this expansion:

\[ \Delta^{-1} = Y D Y^T \quad ; \quad D \quad \text{filter} \]

Leading to the following reformulation of the EEG measurement model:

\[ v(t) = K M p(t) + e(t) = K \tilde{M} \tilde{p}(t) + e(t) \]
The Local Nunez model exhibits highly damped activity spreading out from an occipital source: duality between wave and localized activity.

The estimation procedure evolves from global wave solution to local dynamics.
Detecting EBC must follow a process

1) Model specification
   State Space Model (SSM)

   1.a) Selection of nodes and edges

   1.b) Observation Equation (Table II)

   1.c) State Equations (Table III)

   1.d) Priors for parameters, states and Connectivity

   1.e) Identifiability?

   Type of stochastics

   Type of dynamics

2) Model inversion

   2.a) Discretisation

   2.b) Transformation of Coordinates

   2.c) Model Fit and Assessment

3) Model inference

   3.a) Structural Causal Modeling

   3.b) WAGS Influence

   3.c) Dynamical Structural Systems

   Role of Interventions

Fig. 1. Overview of causal modeling in Neuroimaging. Overall view of conceptual framework for defining and detecting effective connectivity in Neuroimaging studies.
**General theory of Brain Connectivity**

The many levels of causal brain network discovery. Bressler & Mannino. *Physics of Life Review*

Causal Time Series Analysis of functional Magnetic Resonance Imaging Data. Roebroeck, Seth. *JMLR*

Effective connectivity: Influence, causality and biophysical modelling. Friston & Danizeau. *Neuroimage*

Introduction: multimodal neuroimaging of brain connectivity. Friston & Kotter. *PTRS*

**Image Fusion**

Tensor Analysis and Fusion of Multimodal Brain Images. Karahan, Valdes, Bringas. *IEEE*

Incorporating priors for EEG source imaging and connectivity analysis. Xu Lei. *Frontiers*

Model driven EEG/fMRI fusion of brain oscillations. Ozaki, Bosch, Riera, Iturria, Sotero, Aleman, Carbonell. *Hum Brain Map*

Concurrent EEG / fMRI analysis by multiway Partial Least Squares. Martinez, Miwakeichi, Goldman, Cohen. *Neuroimage*

A symmetrical Bayesian model for fMRI and EEG/MEG image fusion. Trujillo, Martinez, Melie. *Int J Bioelect*

**EEG Source Connectivity**

Critical comments on EEG sensor space dynamical connectivity analysis. Van De Steen, Faes, Karahan, Songsiri, Marinazzo. *BTOP*

Cortical current source connectivity by means of partial coherence fields. Pascual, Riera. *ArXiv*

White matter architecture rather than cortical surface area correlates with the EEG alpha rhythm. Valdes Hdez, Ojeda, Martinez, Lage, Virues. *Neuroimage*

Minimum Overlap Component Analysis (MOCA) of EEG/MEG data for more than two sources. Nolte, Marzetti. *J Neurosc Meth*

Decomposing EEG data into space-time-frequency components using Parallel Factor Analysis. Miwakeichi, Martinez Yamaguchi. *Neuroimage*

**Extension to Brain Body network**

Gait Influence Diagrams in Parkinson’s Disease. Ren, Karahan ... Bosch, Bringas. *IEEE*
<table>
<thead>
<tr>
<th>Collaborators</th>
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CCC China Cuba Canada alliance for Neuroinformation
Dezhong Yao, Bharat Biswal and Pedro Valdes are recruiting:

Professors
Postdocs
PhD students

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