M/EEG Decoding and Brain-Computer Interfacing

Moritz Grosse-Wentrup

Max Planck Institute for Intelligent Systems
Department Empirical Inference
Tübingen, Germany

June 8, 2014





Brain-Computer Interfacing in ALS





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Outline

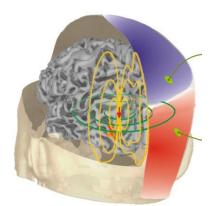
M/EEG Decoding Models

2 Brain-Computer Interfacing

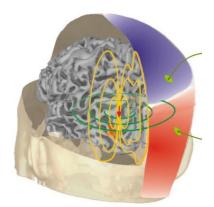
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M/EEG Decoding Models

Brain-Computer Interfacing

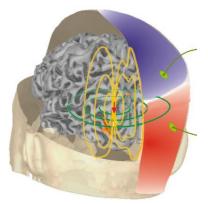


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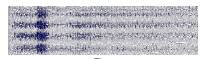
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Time

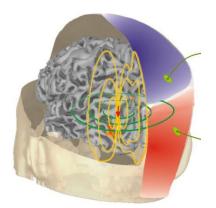


Notation:

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Time



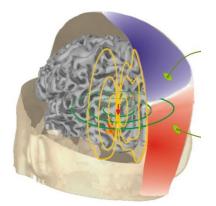
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Notation:

• M/EEG signal $\mathbf{x}_i[t] \in \mathbb{R}^N$



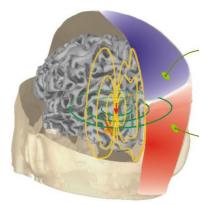
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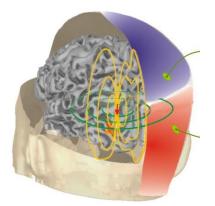
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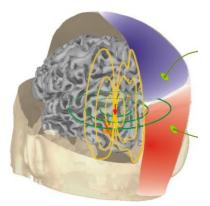
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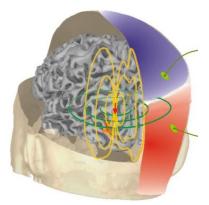
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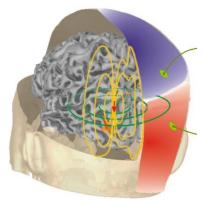
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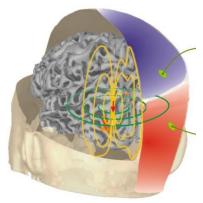
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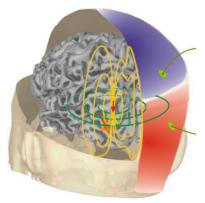
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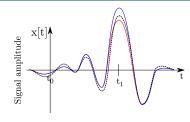
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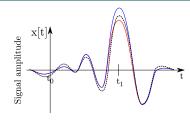
- $\bullet \ (c_i, X_i) \sim p(c, X)$
- $\mathcal{D} = \{(c_1, X_1), \ldots, (c_M, X_M)\}$
- Typically i.i.d. sampling is assumed

• Given a certain brain-state, what is the probability of an experimental condition: p(c|X)?

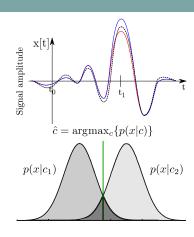
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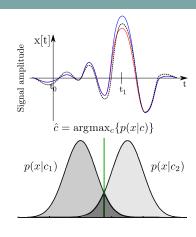
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- The (optimal) Bayes classifier: $\hat{c}_i = f(X_i) = \operatorname{argmax}_c\{p(X_i|c)\}$



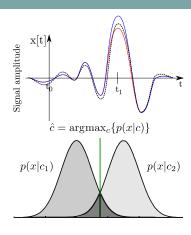
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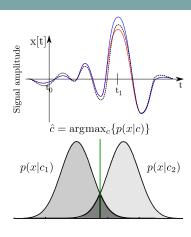
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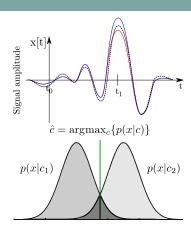
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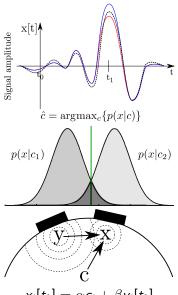
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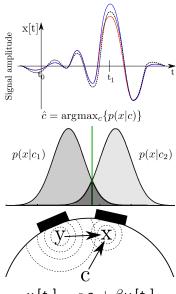


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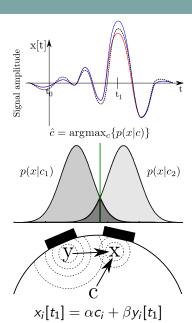


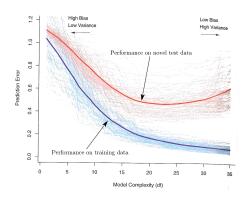
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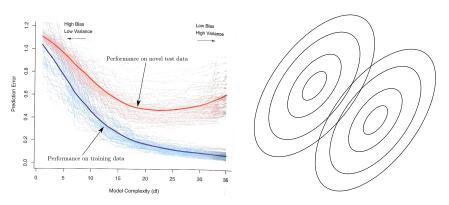
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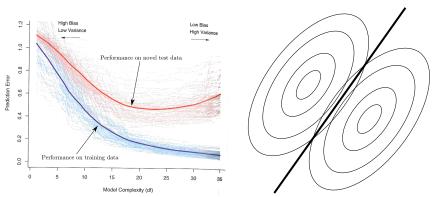


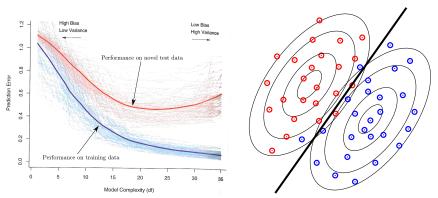
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- Disadvantage $f : \mathbb{R}^{N \times T} \mapsto \{-1, +1\}$ needs to be learned from \mathcal{D} .

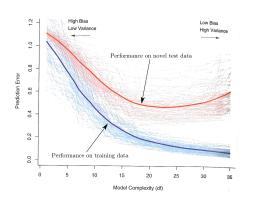


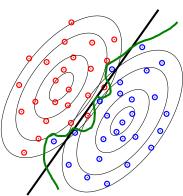




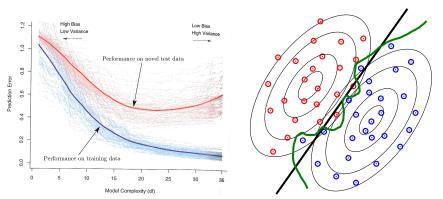






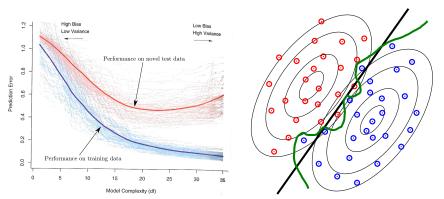


The decoder f has to be chosen from a model class \mathcal{F} . How to choose \mathcal{F} ?



How do we determine which model generalizes best?

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How do we determine which model generalizes best? Cross-validation!



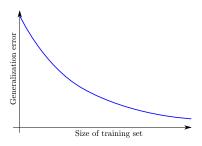
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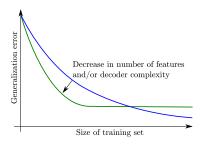
the model complexity,

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- and the number of features $N \times T$.

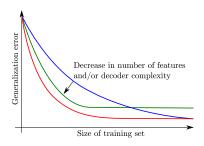
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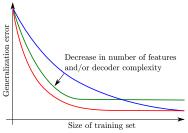


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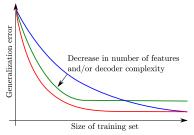
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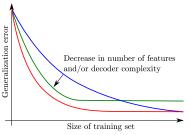


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• reduce N and T without discarding information relevant for c,

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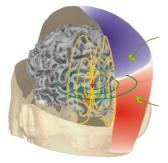
- reduce N and T without discarding information relevant for c,
- and find a *simple* representation of X.

(Hastie, Tibshirani, & Friedman. The Elements of Statistical Learning. Springer, 2009)



$$x[t] = Ls[t]$$

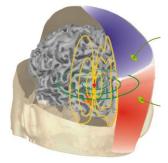
- ullet Source vector $\mathbf{s}[t] \in \mathbb{R}^K$
- Leadfield matrix $L \in \mathbb{R}^{N \times K}$



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$$y[t] = \mathbf{w}^\mathsf{T} \mathbf{x}[t] = \mathbf{w}^\mathsf{T} L \mathbf{s}[t]$$

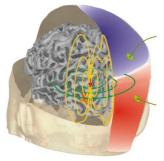
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$$y[t] = \mathbf{w}^\mathsf{T} \mathbf{x}[t] = \mathbf{w}^\mathsf{T} L \mathbf{s}[t] = \mathbf{g}^\mathsf{T} \mathbf{s}[t]$$

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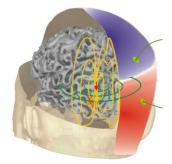


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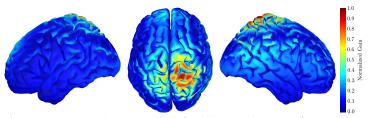
Spatial filtering of M/EEG data:

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(Baillet, Mosher & Leahy. Electromagnetic brain mapping. IEEE Signal Processing Magazine, 2001)

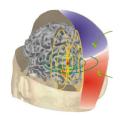
Unsupervised method based on a-priori knowledge of the spatial origin of relevant sources:

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Pick a cortical target source s*

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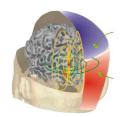
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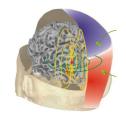
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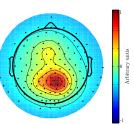
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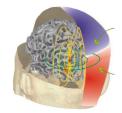


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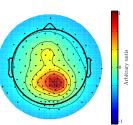


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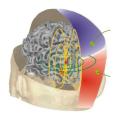


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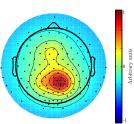


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- Solve the optimization problem $\mathbf{w} = \operatorname{argmin}_{\mathbf{w}} \{ \mathbf{w}^{\mathsf{T}} \mathbf{\Sigma} \mathbf{w} \} \text{ s.t. } \mathbf{w}^{\mathsf{T}} \mathbf{a} = 1$

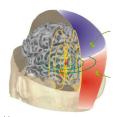


http://www.canada-meg-consortium.org/

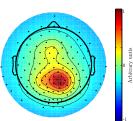


Unsupervised method based on a-priori knowledge of the spatial origin of relevant sources:

- Pick a cortical target source s*
- Compute the forward solution $\mathbf{a} = \mathbf{I}s^*$
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- $\bullet \ \mathbf{w} = \mathbf{a}^\mathsf{T} \Sigma^{-1} / (\mathbf{a}^\mathsf{T} \Sigma^{-1} \mathbf{a})$

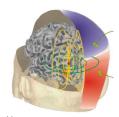


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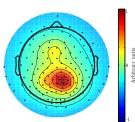


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- Check the gain vector $\mathbf{g} = \mathbf{w}^{\mathsf{T}} L$

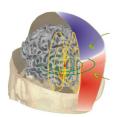


http://www.canada-meg-consortium.org/

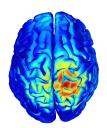


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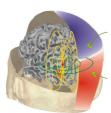


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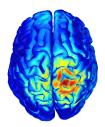
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M. Grosse-Wentrup (MPI-IS)

- Check the gain vector $\mathbf{g} = \mathbf{w}^{\mathsf{T}} L$
- Apply the spatial filter: $y[t] = \mathbf{w}^{\mathsf{T}}\mathbf{x}[t]$



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Supervised method to find spatial filters that discriminate between two conditions:

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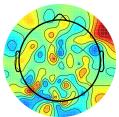
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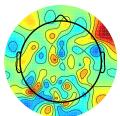
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$$\sum_{c=+1}^{\infty} \sum_{c=-1}^{\infty} \mathbf{w} = \lambda \mathbf{w}$$

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Supervised method to find spatial filters that discriminate between two conditions:

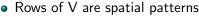
- Covariance matrices $\Sigma_{c=+1} \& \Sigma_{c=-1}$
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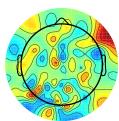
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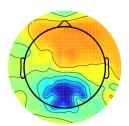
• Solution: Largest λ for which $\sum_{c=+1}^{-1} \sum_{c=-1} \mathbf{w} = \lambda \mathbf{w}$

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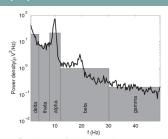
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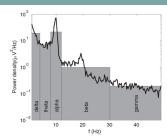
(Ramoser et al. Optimal spatial filtering of single trial EEG during imagined hand movement. IEEE TBME, 2000)



(van Albada & Robinson, Frontiers in Human Neuroscience, 2013)

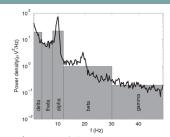
The brain cares about oscillations:

• DTFT $(y_i[t], \omega) = 1/T \sum_{t=0}^{T-1} y_i[t]e^{-j\omega t}$



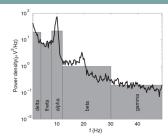
(van Albada & Robinson, Frontiers in Human Neuroscience, 2013)

- DTFT $(y_i[t], \omega) = 1/T \sum_{t=0}^{T-1} y_i[t]e^{-j\omega t}$
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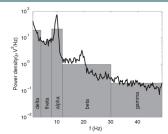
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- $\mathcal{Z}_i = \{z_i[\delta], z_i[\theta], z_i[\alpha], z_i[\beta], z_i[\gamma]\}$

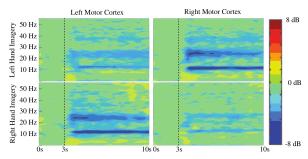


(van Albada & Robinson, Frontiers in Human Neuroscience, 2013)

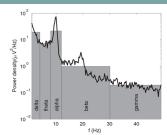
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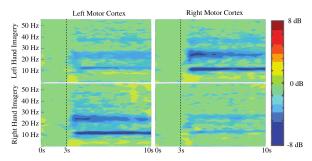
(van Albada & Robinson, Frontiers in Human Neuroscience, 2013)



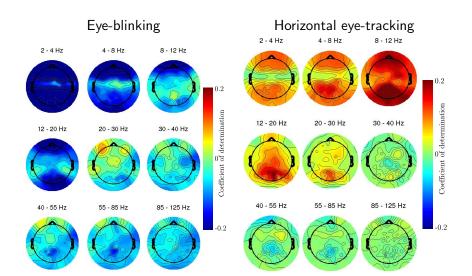
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- For log-bandpower features, linear decoders appear sufficient.



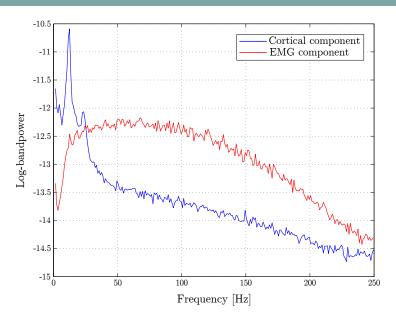
(van Albada & Robinson, Frontiers in Human Neuroscience, 2013)



Confounding by EOG-artifacts



Confounding by EMG-artifacts

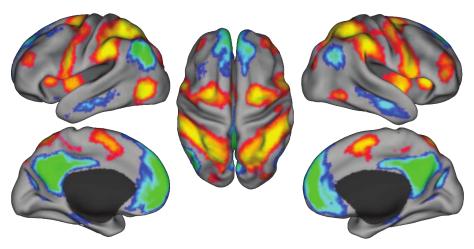


Outline

M/EEG Decoding Models

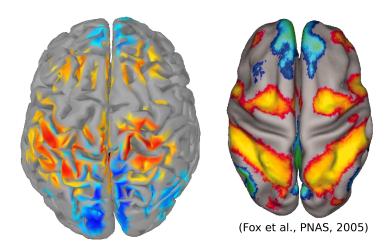
2 Brain-Computer Interfacing

Large-scale brain networks

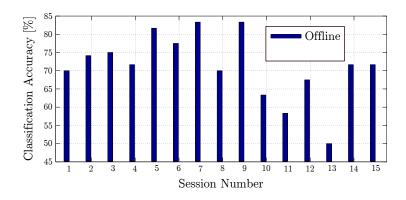


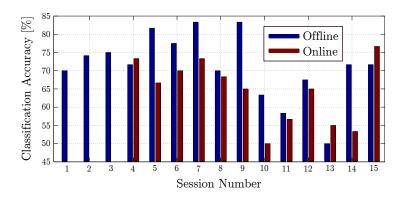
(Adapted from Fox et al., 2005)

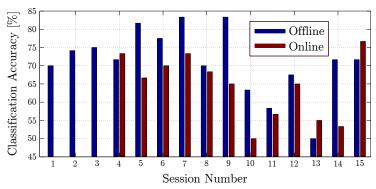
Can large-scale cortical networks be observed in the EEG?



(Grosse-Wentrup & Schölkopf, High Gamma-Power Predicts Performance in SMR BCIs, Journal of Neural Engineering, 2012)

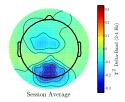




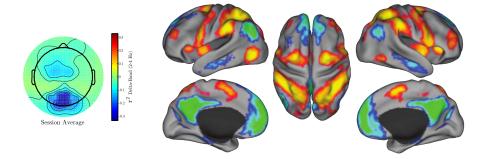


Mean offline/online decoding accuracy: 71.3%/64.4%

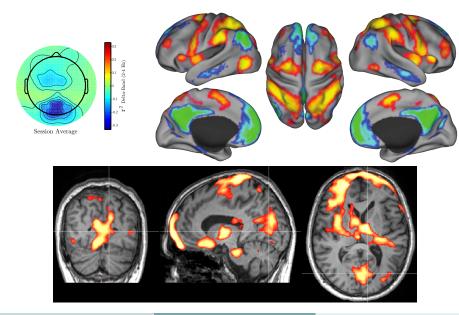
Patient GH: fMRI-study



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Acknowledgements

Max Planck Institutes:

- Bernd Battes
- Alexander Bretin
- Tatiana Fomina
- Christian Förster
- Marius Klug
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- Natalie Widmann
- Bernhard Schölkopf

- Nadine Simon
- Sebastian Weichwald

University of Tübingen:

- Michael Erb
- Thomas Ethofer

http://brain-computer-interfaces.net